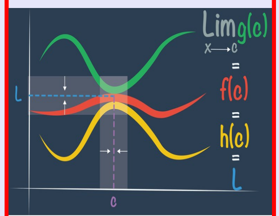
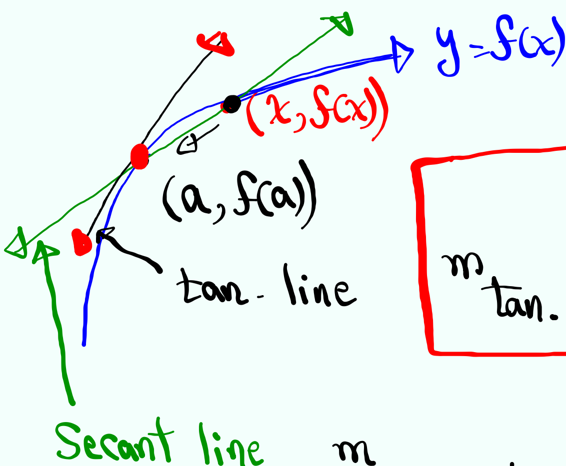
Calculus I

Lecture 16



Feb 19-8:47 AM



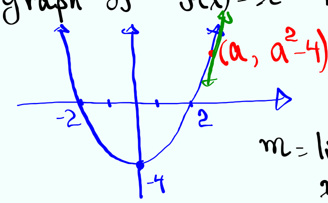
$$\boxed{m_{\text{tan. line}} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}}$$

$$m_{\text{secant line}} = \frac{\Delta y}{\Delta x} = \frac{f(x) - f(a)}{x - a}$$

as $x \to a$ then Secant line becomes tangent line.

Sep 24-7:27 AM

Find slope of the tan. line at $x=a$ to the graph of $f(x)=x^2-4$.



$$m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

$$m=\lim_{x\to a}\frac{x^2\cancel{-4}-a^2\cancel{+4}}{x-a}=\lim_{x\to a}\frac{x^2-a^2}{x-a}\quad \frac{0}{0}\ \text{I.F.}$$

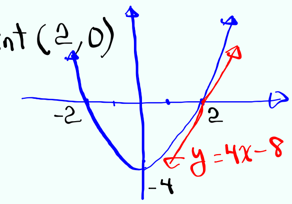
$$=\lim_{x\to a}\frac{(x+a)\cancel{(x-a)}}{\cancel{x-a}}=\lim_{x\to a}(x+a)=a+a=\boxed{2a}$$

If $a=2$, $m=4$, Point $(2,0)$

$y-y_1=m(x-x_1)$

$y-0=4(x-2)$

$\boxed{y=4x-8}$



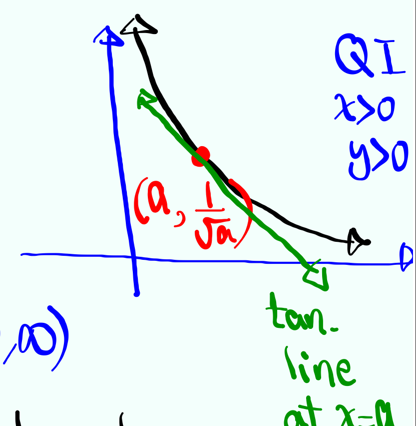
Sep 24-7:33 AM

Find slope of the tan. line to the graph of $f(x)=\frac{1}{\sqrt{x}}$ at $x=a$.

$x>0$ Domain $(0,\infty)$

$\sqrt{x}\geq 0$

$\frac{1}{\sqrt{x}}>0$ $y>0$ Range $(0,\infty)$



$$m=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}=\lim_{x\to a}\frac{\frac{1}{\sqrt{x}}-\frac{1}{\sqrt{a}}}{x-a}\quad \frac{0}{0}\ \text{I.F.}$$

Sep 24-7:40 AM

$$m = \lim_{x \to a} \frac{\frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a}}}{x - a} = \lim_{x \to a} \frac{\sqrt{a} - \sqrt{x}}{\sqrt{x}\sqrt{a}(x-a)} \quad \frac{0}{0} \text{ I.F.}$$

$LCD = \sqrt{x}\sqrt{a}$

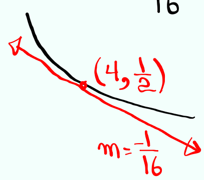
$$= \lim_{x \to a} \frac{(\sqrt{a} - \sqrt{x})(\sqrt{a} + \sqrt{x})}{\sqrt{x}\sqrt{a}(x-a)(\sqrt{a}+\sqrt{x})}$$

$$\frac{a-b}{b-a} = -1$$

$$= \lim_{x \to a} \frac{\overset{-1}{\cancel{a - x}}}{\sqrt{x}\sqrt{a}\cancel{(x-a)}(\sqrt{a}+\sqrt{x})}$$

$$= \lim_{x \to a} \frac{-1}{\sqrt{x}\sqrt{a}(\sqrt{a}+\sqrt{x})} = \frac{-1}{\sqrt{a}\sqrt{a}(\sqrt{a}+\sqrt{a})} = \frac{-1}{a \cdot 2\sqrt{a}} = \boxed{\frac{-1}{2a\sqrt{a}}}$$

So $a = 4$, $m = \frac{-1}{16}$, $(4, \frac{1}{2})$



$$y - \frac{1}{2} = \frac{-1}{16}(x - 4)$$

$$y = \frac{-1}{16}x + \frac{1}{4} + \frac{1}{2}$$

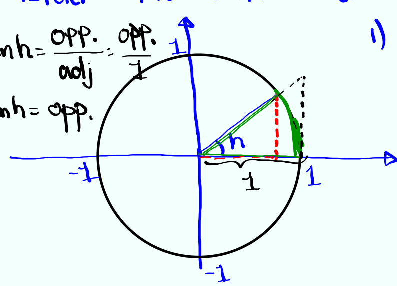
$$\boxed{y = \frac{-1}{16}x + \frac{3}{4}}$$

Sep 24-7:45 AM

Consider the unit circle below

$\tan h = \frac{opp.}{adj} = \frac{opp.}{1}$

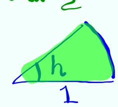
$\tan h = opp.$

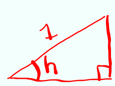


1) Draw a sector with central angle h.

Area of Sector

$Area = \frac{1}{2}r^2\theta$

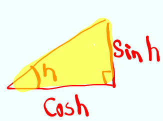




$\sin h = \frac{opp.}{hyp.} = \frac{opp.}{1} = opp.$

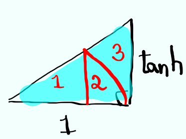
$\cos h = \frac{adj}{hyp.} = \frac{adj.}{1} = adj.$

$Area = \frac{1}{2}(1)^2 h$



$Area = \frac{bh}{2} = \frac{\sin h \cos h}{2}$

$Area = \frac{h}{2}$



$Area = \frac{bh}{2} = \frac{1 \tan h}{2}$

Sep 24-7:55 AM

Area < Area < Area

$$\frac{\sinh\cosh}{2} < \frac{h}{2} < \frac{\tan h}{2}$$

LCD = 2

$$\sinh\cosh < h < \tan h$$

$$\sinh\cosh < h < \frac{\sinh}{\cosh}$$

Divide by $\sinh > 0$ in QI & QII

$$\cosh < \frac{h}{\sinh} < \frac{1}{\cosh}$$

Do reciprocal

$$\frac{1}{\cosh} > \frac{\sinh}{h} > \cos h$$

$$\cosh < \frac{\sinh}{h} < \frac{1}{\cosh}$$

$\lim_{h\to 0} \cosh = 1$

$\lim_{h\to 0} \frac{1}{\cosh} = \frac{1}{1} = 1$

$\lim_{h\to 0} \frac{\sinh}{h} = 1$ by S.T.

Sep 24-8:07 AM

Evaluate $\lim_{h\to 0} \frac{4\sin 4h}{4h}$

Recall $\lim_{h\to 0} \frac{\sinh}{h} = 1$

$$= 4\left(\lim_{h\to 0} \frac{\sin 4h}{4h}\right) = 4\cdot 1 = \boxed{4}$$

Evaluate $\lim_{x\to -2} \frac{\sin(x+2)}{x^3+8}$ $\frac{0}{0}$ I.F.

$$\lim_{x\to -2} \frac{\sin(x+2)}{x^3+8} = \lim_{x\to -2} \frac{\sin(x+2)}{(x+2)(x^2-2x+4)}$$

$$= \lim_{x\to -2}\left[\frac{\sin(x+2)}{x+2}\cdot\frac{1}{x^2-2x+4}\right]$$

$$= \lim_{x\to -2}\frac{\sin(x+2)}{x+2}\cdot\lim_{x\to -2}\frac{1}{x^2-2x+4}$$

$$= 1\cdot\frac{1}{(-2)^2-2(-2)+4} = \boxed{\frac{1}{12}}$$

Sep 24-8:15 AM

Prove $\lim_{h\to 0} \frac{\cos h - 1}{h} = 0$

$$\lim_{h\to 0} \frac{\overset{-1(1-\cos h)}{\cos h - 1}}{h} = \lim_{h\to 0} \frac{-1(1-\cos h)}{h}$$

$$= -1 \cdot \lim_{h\to 0} \frac{1-\cos h}{h} = -\lim_{h\to 0} \frac{1-\cos h}{h} \cdot \frac{1+\cos h}{1+\cos h}$$

$$= -\lim_{h\to 0} \frac{1-\cos^2 h}{h(1+\cos h)} = -\lim_{h\to 0} \frac{\sin^2 h}{h(1+\cos h)}$$

$$= -\lim_{h\to 0} \left[\frac{\sin h}{h} \cdot \frac{\sin h}{1+\cos h} \right]$$

$$= -\lim_{h\to 0} \frac{\sin h}{h} \xrightarrow{} 1 \cdot \lim_{h\to 0} \frac{\sin h}{1+\cos h}$$

$$= -\frac{\sin 0}{1+\cos 0} = -\frac{0}{1+1} = \frac{0}{2} = \boxed{0}$$

$$\boxed{\lim_{h\to 0} \frac{\cos h - 1}{h} = 0}$$

Sep 24-8:22 AM